$P \vee Q \qquad \neg P \vee R$

$\Rightarrow_{Res} \quad Q \vee R$ apply only once

$$\frac{P \vee Q \qquad \neg P \vee Q \vee P}{Q \vee Q \vee P}$$

$\Rightarrow_{RES}$

$\Rightarrow_{RES} \quad Q \vee Q \vee Q \vee P$

No dup literals no tautologies 3^n

n variables

$$N = \{\underline{P \vee Q},\ \underline{\neg P \vee Q},\ \underline{P \vee \neg Q},\ \underline{\neg P \vee \neg Q}\}$$

$$(\varepsilon, N, \emptyset, 0, \top)$$

$$\xRightarrow[\text{CDCL}]{\text{Decide}} ([P^{\wedge}], N, \emptyset, 1, \top)$$

$$\neg P \vee Q \qquad \neg P \vee \neg Q \qquad \xRightarrow[\text{RES}]{\text{Cond}} \neg P$$

$$\xRightarrow[\text{CDCL}]{\text{Propagate}} ([P^{\wedge} Q^{\neg P \vee Q}], N, \emptyset, 1, \top)$$

$$\xRightarrow[\text{CDCL}]{\text{Conflict}} ([P^{\wedge} Q^{\boxed{\neg P \vee Q}}], N, \emptyset, 1, \neg P \vee \neg Q)$$

$$\xRightarrow[\text{CDCL}]{\text{Resolve}} ([P^{\wedge}], N, \emptyset, 1, \neg P)$$

$$\xRightarrow[\text{CDCL}]{\text{Backtrack}} ([\neg P^{\neg P}], N, \{\neg P\}, 0, \top)$$

$$\xRightarrow[\text{CDCL}]{\text{Propagate, Conflict}} ([\neg P^{\neg P} Q^{\underline{P \vee Q}}, N, \{\neg P\}, 0, \underline{P \vee \neg Q})$$

$$\xRightarrow[\text{CDCL}]{\text{Resolve}} (\neg P^{\neg P}, N, \{\neg P\}, 0, P) \Longrightarrow (\ldots, \bot)$$

Proof

1. M is consistent

Induction length of $\Longrightarrow^{+}_{CDCL}$ derivation

base case: $(\varepsilon, N, \emptyset, 0, \top)$

ε consistent

induction case:

assume (M, N, U, k, D)

M consistent

$\Longrightarrow_{CDCL} (M', N, U', k', D')$

show M' consistent

$M \not\ni M'$ and $M' \not\ni M$

Propagate Decide $(M \; \ldots\ldots)$ L undefined

$\Longrightarrow (ML, \ldots\ldots)$

2. all learned clauses entailed

$$\xRightarrow[\text{CDCL}]{\text{Conflict}} (M, N, U, k, D) \xRightarrow[\text{CDCL}]{\text{Skip/Resolve}}{}^{*} (M', N, U, k, D')$$

$$D \notin \{\bot, \top\}$$

$$\xRightarrow[\text{CDCL}]{\text{Backtrack}} (M'', N, U \cup \{D'\}, i, \top)$$

$$N \cup U \models D'$$

base: $D \in (N \cup U) \leadsto N \cup U \models D$

Skip ✓

Resolve: Resolution $\leadsto$ sound

$$(M L^{C \vee L}, \ldots, D'' \vee \text{comp}(L)) \checkmark$$

$$N = \{ \underline{\neg P \lor Q}, \quad \underline{\underline{P \lor R}}, \quad Q \lor R, \quad \dots \}$$

$$\underline{\neg P \lor \neg R}$$

$$\Rightarrow_{RES} \quad Q \lor R$$

$$\Rightarrow_{RES} \quad R \lor \neg R$$

Newly learnt clause with unsound strategy
not contained in $N \cup U$

Proof

learn a clause D by unsound rule and
$D' \in (N \cup U)$ $D' = D \vee L$

$(M_1 K^{i+1} M_2, N, U, k, D \vee L)$

$\overset{\text{Backtrack}}{\Longrightarrow} (M_1 L^{D \vee L}_{\top}, N, U, i, \top)$

$D \vee L \in \{N \cup U\}$

Prefer conflict
over Propagation in
rest