$$N = \left\{ \begin{array}{l} \underline{R(x,y) \vee P(a,b)} \\ \neg P(a,b) \\ \neg R(b,b) \end{array} \right\} \text{ unsat}$$

$$N_Q = \left\{ \begin{array}{l} R(a,a) \vee P(a,b) \\ \neg P(a,b) \\ \underline{\neg R(b,b)} \end{array} \right.$$

unsat

$\rightsquigarrow$ $S_1 \vee S_2$, $\neg S_2$, $\neg S_3$

$S_3 \vee S_2$

$[\neg S_3, \neg S_2, S_1 \vee S_2]$

$R(x,y) \vee P(a,b) \cdot \{x \to a, y \to a\}$

$R(x,y) \vee P(a,b) \cdot \{x \to b, y \to b\}$

$R(b,b) \vee P(a,b)$

$N_Q = \{ \underline{\neg R(b,b)}, \underline{\neg P(a,b)}, R(a,a) \}$

$\neg R(b,b)$

$R(b,b) \vee P(a,b)$

min false clause

FOL undecidability ⇝ 1920 - 1940

Post Correspondance Problem ⇝ 1940 <

$(w_1, w_2, \dots, w_n) \quad (v_1, v_2, \dots, v_n)$

$I = [i_1, \dots, i_k] \quad 1 \le i_j \le n$

$w_{i_1} w_{i_2} \dots w_{i_n} = v_{i_1} v_{i_2} \dots v_{i_n}$

$\Sigma = \{a, b\}$ $(a, ab, bba) \quad (ab, ba, ba)$

$I = [1, 3]$

$a\ bba = ab\ ba$

Horn

dec

- $R(\varepsilon, \varepsilon)$
- $\neg R(x, y) \vee R(f_a(x), f_a(f_b(y)))$
- $\neg R(x, y) \vee R(f_a(f_b(x)), f_b(f_a(y)))$
- $\neg R(x, y) \vee R(f_b(f_b(f_a(x))), f_b(f_a(y)))$
- $\neg R(f_a(x), f_a(x))$
- $\neg R(f_b(x), f_b(x))$

non-linea
X occurs twice

Fac

$\Rightarrow_{RES}$

$R(x,b) \vee R(a,y)$

Sat { $R(a,b)$; $\neg R(c,b)$; $\neg R(a,c)$ } unsat

$\sigma = \{x \to a, y \to b\}$

Fac

$\Rightarrow_{Res}$

$R(x,y) \vee R(x',y')$ $\quad \sigma = \{x \to x', y \to y'\}$

$R(x',y')$

Subsumption

C $\subset$ D

$C\sigma \subset D$

Tautology

$\neg P(x) \vee P(f(y))$ not a tautology

$\neg P(x) \vee P(x)$ is a tautology

$\neg P(x) \vee P(y)$ not a tautology

unsat

$P(a)$

$\neg P(f(a))$

sat

Schlussel Resolution

$$R(x) \vee P(f(x))$$

$$\neg P(f(a)) \vee R(a) \vee Q(a)$$

SUBRES

RES

$\sigma = \{x \to a\}$

RES

$R(a) \vee R(a) \vee Q(a)$

RES

$R(a) \vee Q(a)$

CON 1

RES

Subsumes $\neg P(f(a)) \vee R(a) \vee Q(a)$

Subsumption
NP-complete

3-SAT is NP-complete every clause (exactly) 3 literals

2-SAT is in P every clause at most 2 literals

$P \vee Q \vee \neg R \vee T$ ⟷ (fresh S) $P \vee Q \vee S$, $\neg R \vee T \vee \neg S$

$P \vee Q \vee S$ $\neg P \vee R$, $\neg S$

$\leftarrow P_{a \vee}(x, y, z) \vee P_{a \vee}(f_{aux}(x), z', z) \vee \ldots$

$P_{a \vee}(\top, \top, \top) \vee P_{a \vee}(\top, 0, \top) \vee P_{a \vee}(f_{aux}(0), 0, 0)$

Filtering: a sufficient easy to compute criterion that a clause does not subsume

① #vars in a clause $|C| \neq |D|$

$|vars(C)| > |vars(D)|$

then $D\tau \subseteq C$ for some τ

$D = P(a) \qquad C = P(x)$

② number of literals $|D| > |C|$

D cannot subsume C 95% <

(1) $P(a)$

(2) $\neg P(x) \vee P(g(x))$

(3) $\neg P(g(g(a)))$

• 1.1 Res 2.1 (4) $P(g(a))$ $\{x \to a\}$

• 4.1 Res 2.1 (5) $P(g(g(a)))$ $\{x \to g(a)\}$

• 3.1 Res 5.1 (6) $\bot$

Step 3-5 is Sol Res Step (6)

2.1 Res 2.2 (4') $\neg P(x) \vee P(g(g(x)))$ $\{x' \to g(x)\}$

4'.1 Res 4'.2 (4'') $\neg P(x) \vee P(g(g(g(a))))$

4'.1 Res 1.1 (5') $P(g(g(a)))$

3.1 Res 5' (6') $\bot$

Ren $\{x \to x'\}$

↖ self resolution