UNIVERSITÄT DES SAARLANDES
MPI – Informatik
Christoph Weidenbach

Lecture "Automated Reasoning"
(Winter Term 2022/2023)

Midterm Examination

Name: ....

Student Number: ....

Some notes:

- Things to do at the beginning:

  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones and any other electronic device.
  Whenever you use a new sheet of paper (including scratch paper), first write your name and student number on it.

- Things to do at the end:

  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your examination text.
  Note: Sheets that are accidentally taken out of the lecture room are invalid.

Sign here: **Good luck!**

....

Problem	1	2a	2b	2c	3	4	5	6a	6b	6c	7	Σ
Answered?												
Points												

Problem 1 *(Superposition Refutation)* (5 points)

Show unsatisfiability of the below clauses via the superposition calculus including redundancy elimination based on the atom ordering $S \succ P \succ Q \succ R$ via the syntactic restrictions of the inference rules. You do not need to use the partial model operator.

(1) $\neg P \lor Q \lor R$

(2) $\neg S \lor P \lor R$

(3) $S \lor P \lor R$

(4) $\neg R \lor Q$

(5) $\neg P \lor R$

(6) $\neg P \lor \neg Q$

(7) $\neg R \lor P$

Problem 2 *(Superposition Model Building)* (4 + 2 + 2 = 8 points)

Let N be the clause set $\{P \vee Q \vee R,\ \neg P \vee \neg Q,\ \neg Q \vee \neg P \vee R,\ \neg R \vee Q\}$ with $R \succ Q \succ P$.

1. Determine $N_{\mathcal{I}}$.
2. Which clause is false in $N_{\mathcal{I}}$?
3. Show the superposition inference yielding a smaller counterexample.

Problem 3 *(CDCL)* (7 points)

Use CDCL to decide satisfiability of the following clause set.

(1) $P_1 \vee P_2 \vee P_3$	(2) $P_1 \vee \neg P_3$	(3) $\neg P_1 \vee P_4$
(4) $\neg P_1 \vee P_5$	(5) $P_2 \vee \neg P_4 \vee \neg P_5$	(6) $\neg P_2$

Problem 4 *(CNF)* (6 points)

Transform the formula

$$[\neg(\neg P \vee (Q \wedge R))] \rightarrow [P \wedge (\neg\top \leftrightarrow \neg R)]$$

into CNF using $\Rightarrow_{\text{ACNF}}$.

Problem 5 *(Fourier Motzkin)* (6 points)

Use the FM method to decide whether the following conjunction of inequations is satisfiable:

$$\begin{aligned} x + y &\geq 16 \\ 4x + 7y &\leq 28 \\ 2x - 7y &\leq 20 \\ 2x - 3y &\geq -9 \end{aligned}$$

Problem 6 *(Conjectures)* (2 + 2 + 2 = 6 points)

Which of the following statements are true or false? Provide a proof or a counter example.

1. If for some unification problem $E = \{t = s\}$ we have $\text{vars}(t) \cap \text{vars}(s) = \emptyset$ then the size of an eventual mgu (number of symbols) is polynomial in $|s| + |t|$.
2. If for some propositional clause set N we have $N_{\mathcal{I}} \models N$ then for any clause D: $N \models D$ if and only if $N_{\mathcal{I}} \models D$.
3. If CDCL with start state $(\epsilon, N, \emptyset, 0, \top)$ finishes in a state $(M, N, U, k, \top)$ by a reasonable strategy, i.e., $M \models N$, then there is no $M' \subset M$ such that $M' \models N$.

Problem 7 *(First-Order Logic Semantics)* (4 points)

Prove that the formula $\forall x.((P(x) \leftrightarrow Q(x)) \vee Q(g(x)))$ is satisfiable if and only if the formula $(\forall x.(R(x) \vee Q(g(x)))) \wedge \forall x.(R(x) \leftrightarrow (P(x) \leftrightarrow Q(x)))$ is satisfiable where both interpretations agree on the domain and the interpretation of P and Q.