UNIVERSITÄT DES SAARLANDES
MPI – Informatik
Christoph Weidenbach

Lecture “Automated Reasoning”
(Winter Term 2024/2025)

Final Examination

Name: ....

Student Number: ....

Some notes:

- Things to do at the beginning:

  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones and any other electronic device.
  Whenever you use a new sheet of paper (including scratch paper), first write your name and student number on it.

- Things to do at the end:

  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your examination text.
  Note: Sheets that are accidentally taken out of the lecture room are invalid.

Sign here:

Good luck!

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Problem	1	2	3	4	5	6a	6b	6c	7	Σ
Answered?										
Points										

Problem 1 *(First-Order Superposition Refutation)* (8 points)

Refute the below clause set using a KBO, all symbols have weight one and the precedence is $g \succ f \succ T \succ Q \succ R \succ a \succ b$.

(1)	$R(f(x,y),g(b)) \lor R(f(x,y),g(y))$	(2)	$R(f(x,y),g(b)) \lor \neg Q(g(b))$
(3)	$T(y,g(x)) \lor \neg R(g(x),f(x,y))$	(4)	$\neg R(f(x,y),g(b)) \lor R(g(b),f(x,y))$
(5)	$\neg T(x,g(y)) \lor \neg R(g(y),x)$	(6)	$R(g(y),x) \lor Q(g(x))$

Problem 2 *(SCL(FOL))* (6 points)

Refute the following clause set using SCL(FOL), starting with a trail $[P(a,b)^1]$, i.e., $P(a,b)$ is decided to be true and all literals with less than 7 symbols may be considered.

(1) $P(x,y) \lor P(y,x)$

(2) $\neg P(x,y) \lor P(f(y,y),y)$

(3) $\neg P(x,y) \lor Q(g(y))$

(4) $\neg Q(g(x)) \lor P(f(y,y),y)$

(5) $\neg P(f(x,x),x) \lor \neg P(f(x,x),x)$

Problem 3 *(CDCL)* (7 points)

Use CDCL to decide satisfiability of the following clause set.

(1)	$\neg P_1 \vee P_2 \vee P_3$	(2)	$\neg P_4 \vee P_1 \vee P_5$	(3)	$P_4 \vee P_1 \vee P_5$
(4)	$\neg P_5 \vee P_2$	(5)	$\neg P_3 \vee P_5$	(6)	$\neg P_1 \vee \neg P_2$
(7)	$\neg P_3 \vee P_1$				

Problem 4 *(Knuth Bendix Completion)* (6 points)

Apply completion ($\Rightarrow_{\text{KBC}}$) to the following set of equations with respect to a KBO where all signature symbols (and variables) have weight 1 and $f \succ g$ and x, y are variables.

$$E = \{f(x,x) \approx x, f(g(y),y) \approx g(y), g(g(x)) \approx g(x)\}$$

Problem 5 *(Simplex)* (8 points)

Decide satisfiability of the following system of inequations using simplex

$$\begin{aligned} y &\geq x + 2 \\ 2y &\geq -x - 1 \\ 5y &\leq -x + 5 \end{aligned}$$

starting with equations and simple bounds

$$\begin{array}{ll} z_1 = y - x & z_1 \geq 2 \\ z_2 = 2y + x & z_2 \geq -1 \\ z_3 = 5y + x & z_3 \leq 5 \end{array}$$

where you propagate bounds in the order z_1, z_2, z_3 and select for pivoting first x, then y, and then the other variables in the above order.

Problem 6 *(Conjectures)* (2 + 2 + 2 = 6 points)

Which of the following statements are true or false? Provide a proof or a counter example.

a) In Knuth-Bendix Completion, any critical pair of a rewrite rule $t \rightarrow s$ with itself can eventually be deleted.

b) A simplex problem of the form $c_i \leq \sum a_{i,j} x_{i,j}$ where $0 \leq c_i$ for all c_i (and so no $\geq$) does always have a solution.

c) First-order superposition can be turned (by choosing an appropriate ordering and selection strategy) into a decision procedure (it terminates) for any Horn clause set (Horn means at most one positive literal in a clause) where all positive literals in unit clauses (clauses with exactly one literal) are ground.

Problem 7 *(KBC)* (4 points)

Let $E = \{t_i \approx s_i \mid 1 \leq i \leq n\}$ be a finite set of equation such that $\text{vars}(t_i) = \text{vars}(s_i)$ and all t_i, s_i are of the form $f(x_1, \ldots, x_n)$, where t_i, s_i may have different top function symbols, with $x_i < x_{i+1}$ for some total strict ordering $<$ on the variables. Prove: KBC terminates on E resulting in a convergent system.