UNIVERSITÄT DES SAARLANDES

MPI – Informatik
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Lecture "Automated Reasoning"
(Winter Term 2024/2025)

Final Examination Test

Name: ....

Student Number: ....

Some notes:

- Things to do at the beginning:

  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones and any other electronic device.
  Whenever you use a new sheet of paper (including scratch paper), first write your name and student number on it.

- Things to do at the end:

  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your examination text.
  Note: Sheets that are accidentally taken out of the lecture room are invalid.

Sign here: **Good luck!**

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Problem	1	2a	2b	2c	3	4	5	6a	6b	6c	7	Σ
Answered?												
Points												

Problem 1 *(First-Order Superposition Refutation)* (8 points)

Refute the following set of clauses via superposition. You may freely choose an ordering and selection function and apply the well-known simplification rules. As usual variables in different clauses are different, x, y denote variables and f, g functions and a, b are constants.

$$
\begin{array}{ll}
1 & R(a, b) \lor R(b, a) \\
2 & \neg R(f(x, y), b) \lor R(b, f(x, y)) \\
3 & \neg R(a, x) \lor R(f(x, x), x) \\
4 & \neg R(b, x) \lor Q(g(x)) \\
5 & \neg Q(g(x)) \lor R(f(y, y), b) \\
6 & \neg R(y, b) \lor \neg R(b, y)
\end{array}
$$

Problem 2 *(SCL(FOL))*

Refute the following clause set using SCL(FOL).

1	$\neg Q(x) \vee \neg R(x, g(x))$	2	$\neg P(x) \vee R(a, x)$	3	$P(g(x))$
4	$\neg P(g(x)) \vee R(x, x)$	5	$S(a)$	6	$\neg S(x) \vee Q(x)$

Problem 3 *(CDCL)* (7 points)

Use CDCL to decide satisfiability of the following clause set.

(1) $P_1 \lor P_2 \lor P_3$ (2) $P_4 \lor P_5 \lor P_6$ (3) $\lnot P_3 \lor P_5$

(4) $\lnot P_1 \lor \lnot P_4$ (5) $\lnot P_2 \lor \lnot P_5$ (6) $\lnot P_3 \lor \lnot P_6$

(7) $\lnot P_6$ (8) $\lnot P_5 \lor P_2$

Problem 4 *(Knuth Bendix Completion)* (6 points)

Apply completion ($\Rightarrow_{\text{KBC}}$) to the following set of equations with respect to a KBO where all signature symbols (and variables) have weight 1 and $f \succ g \succ b \succ a$.

$$E = \{f(g(x), x) \approx f(x, x), f(g(a), b) \approx f(b, a), g(g(x)) \approx g(x)\}$$

Problem 5 *(Simplex)* (6 points)

Decide satisfiability of the following system of inequations using simplex.

$$y \geq x + 1$$
$$2y \geq -x - 1$$
$$5y \leq -x + 5$$

Problem 6 *(Conjectures)* $(2 + 2 + 2 = 6$ points$)$

Which of the following statements are true or false? Provide a proof or a counter example.

a) Surprise.

b) Surprise.

c) Surprise.

Problem 7 *(CDCL(T))* (4 points)

Prove for some of the properties of a sound state that they are preserved during a derivation.