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Tutorials for "Automated Reasoning WS24/25"
Exercise sheet 7

Exercise 7.1:
Let $\Sigma = (\{f, g, h, b\}, \emptyset)$; let R be the term rewrite system

$$\{\, f(x) \to g(x),\ g(x) \to f(b),\ h(x) \to b \,\}.$$

Characterize those Σ-terms that do not have a normal form with respect to R.

Exercise 7.2:
Prove that the following term rewrite system is confluent:

$$\begin{array}{rcl} \{\, f(g(x)) & \to & x \\ g(f(x)) & \to & x \\ f(b) & \to & c \\ b & \to & g(c) \,\} \end{array}$$

Exercise 7.3:
Apply the Knuth-Bendix procedure to the set of equations

$$E = \{\, f(f(x)) \approx g(x),\ f(a) \approx b \,\}$$

and transform it into a finite convergent term rewrite system; use the Knuth-Bendix ordering with weight 1 for all function symbols and variables and the precedence $g \succ f \succ a \succ b$.

Exercise* 7.4:
Find a signature Σ containing at least one constant symbol, a set E of Σ-equations, and two terms $s, t \in \mathcal{T}(\Sigma, \mathcal{X})$ such that

$$\mathcal{T}(\Sigma, \{x_1\})/E \ \models\ \forall \vec{x}(s \approx t),$$

but

$$\mathcal{T}(\Sigma, \{x_1, x_2\})/E \ \not\models\ \forall \vec{x}(s \approx t)$$

where $\vec{x}$ consists of all the variables occurring in s and t. The variables in $\vec{x}$ need not be contained in $\{x_1, x_2\}$.

It is not encouraged to prepare joint solutions, because we do not support joint exams.