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Tutorials for "Automated Reasoning WS24/25"
Exercise sheet 5

Exercise 5.1:
Compute an mgu for the following unification problems using both $\Rightarrow_{\mathrm{SU}}$ and $\Rightarrow_{\mathrm{PU}}$ where x, y ,z and their primed versions are all variables and there is only one sort:

1. $\{f(x, h(x, y)) = f(f(y, z), h(y, z'))\}$
2. $\{h(x, y) = z, g(f(x, x)) = z', g(g(f(a, y))) = g(z')\}$
3. $\{h(x, y) = h(x', y'), y' = f(x, a), f(g(a), z) = y\}$

Exercise 5.2:
Consider the below clause set N, $\Sigma = (\{S\}, \{g, b, a\}, \{P, R\})$, with a KBO ordering where all signature symbols and variables have weight 1 and atoms are compared like terms with precedence $P \succ R \succ g \succ b \succ a$.

1	$\neg P(x) \lor P(g(x))$
2	$\neg P(x) \lor R(x, g(x))$
3	$P(a) \lor P(b)$
4	$\neg R(b, g(b)) \lor P(a)$

1. Compute $\mathrm{grd}(\Sigma, N)_{\mathcal{I}}^{\prec \neg R(b,g(b)) \lor P(b)}$, i.e., generate all ground instances of N smaller than $\neg R(b, g(b)) \lor P(b)$ and run the partial model operator.
2. Determine the minimal false ground clause and its productive counterpart and perform the superposition inference step on the respective first-order clauses from N, not on the ground instances, resulting in the clause set N'.
3. Run the partial model operator on $\mathrm{grd}(\Sigma, N')^{\prec \neg R(b,g(b)) \lor P(b)}$. Can the resulting partial model be extended to a model for N' by adding further (arbitrarily chosen) ground atoms? If no, provide an argument why there is always at least one false clause for any extension, if yes provide the complete model and give an argument why it is a model.

4. Consider the above three steps once more after adding the clause $\neg P(g(b))$ to N.

Exercise 5.3:
Refute the following set of clauses by superposition, including all redundancy rules. You can freely choose an ordering and selection function. As usual one sort for everything and x, y, z are all variables.

1	$\neg R(x,y) \lor \neg R(y,z) \lor R(x,z)$	2	$\neg R(x,x)$	3	$R(x, g(x))$
4	$\neg R(x,y) \lor R(y,x)$				

Exercise* 5.4:
Prove that $\Rightarrow_{\mathrm{PU}}$ terminates.

It is not encouraged to prepare joint solutions, because we do not support joint exams.