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Tutorials for "Automated Reasoning WS24/25"
Exercise sheet 4

Exercise 4.1:
Let $\Sigma = (\{f,\, g,\, h,\, b,\, c\}, \emptyset)$ with g arity 2, f and h arity 1 and b and c constants. and let

$$
\begin{aligned}
t_1 &= g(h(x), h(c)), \\
t_2 &= g(x, x), \\
t_3 &= g(b, f(x)), \\
t_4 &= f(g(x, y)), \\
t_5 &= h(g(x, c)).
\end{aligned}
$$

Determine for each $1 \le i < j \le 5$ whether t_i and t_j are incomparable or comparable (and if so, which term is larger) with respect to

1. a lexicographic path ordering with precedence $f \succ g \succ h \succ b \succ c$,

2. a Knuth-Bendix-ordering with precedence $h \succ f \succ g \succ b \succ c$, where h has weight 1 and all other symbols have weight 2.

Exercise 4.2:
Refute the following set N of clauses

$$
\begin{array}{ll}
(1) \quad P(a,b) \vee P(b,a) & (2) \quad \neg P(a,b) \vee P(f(b,b),b) \\
(3) \quad \neg P(b,a) \vee Q(g(a)) & (4) \quad \neg Q(g(a)) \vee P(f(b,b),b) \\
(5) \quad \neg P(f(b,b),b) \vee \neg P(f(b,b),b) &
\end{array}
$$

both using KBO and LPO with ground superposition by only applying the inference rules
Superposition Left and Factoring:

1. using KBO where all variables and signature symbols have weight 1 and $Q \succ P \succ f \succ g \succ b \succ a$,

2. using LPO with precedence $Q \succ P \succ f \succ g \succ b \succ a$.

Exercise 4.3:
Consider again the above clause set from Exercise 4.2. This time compute the model $N_{\mathcal{I}}$ both for KBO and LPO:

1. using KBO where all variables and signature symbols have weight 1 and $Q \succ P \succ f \succ g \succ b \succ a$. Compute $N_{\mathcal{I}}$, determine the minimal false clause, perform the respective ground superposition inference, add the result to N yielding N' and compute again $N'_{\mathcal{I}}$,

2. using LPO with precedence $Q \succ P \succ f \succ g \succ b \succ a$. Compute $N_{\mathcal{I}}$, determine the minimal false clause, perform the respective ground superposition inference, add the result to N yielding N' and compute again $N'_{\mathcal{I}}$.

It is not encouraged to prepare joint solutions, because we do not support joint exams.