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Tutorials for "Automated Reasoning WS24/25"
Exercise sheet 2

Exercise 2.1:
Use CDCL to decide satisfiability of the following clause set.

(1)	$P_1 \lor P_2 \lor P_3 \lor P_4$	(2)	$\neg P_1 \lor \neg P_2$	(3)	$\neg P_2 \lor \neg P_3$
(4)	$\neg P_1 \lor \neg P_4$	(5)	$\neg P_4 \lor P_1$	(6)	$\neg P_4 \lor P_3$
(7)	$\neg P_3 \lor P_2$	(8)	$\neg P_2 \lor P_3$	(9)	$\neg P_1 \lor P_4 \lor P_3$

Exercise 2.2:
Let $a :\to S$ and $R \subseteq S \times T$. Complete the sort information for g, f, P and variables x, y such that the following formula is well-sorted: $\forall x, y.(R(x, g(x)) \to (f(g(x), a) \approx y \lor P(y) \lor R(x, y)))$

Exercise 2.3:
Which of the following closed formulas are valid, satisfiable, unsatisfiable? Justify, i.e. either prove that the formula is valid or unsatisfiable, or give examples to show that they are satisfiable but not valid.

1. $\forall x.P(x) \to \exists x.P(x)$
2. $\forall x.(P(x) \to P(f(x))) \land P(b) \land \neg P(f(f(b)))$
3. $[\forall x.(P(x) \to P(f(x))) \land P(b)] \to \forall x.P(x)$
4. $[\forall x.(P(x) \to P(f(x)))] \to \exists x.P(x)$
5. $\forall x.(P(x) \lor Q(x)) \to [\forall x.P(x) \lor \forall x.Q(x)]$
6. $\forall x.\exists y.P(x, y) \to \exists x.\forall y.P(x, y)$
7. $\forall x.\,(P(x) \to P(f(x))) \to \forall x.P(x)$

Exercise* 2.4:
Prove or refute the following statements:

1. If ϕ is a first-order formula and x a variable, then ϕ is unsatisfiable if and only if $\exists x.\phi$ is unsatisfiable.
2. If ϕ and ψ are first-order formulas and x is a variable, then $\forall x.(\phi \wedge \psi) \models (\forall x.\phi) \wedge (\forall x.\psi)$ and $(\forall x.\phi) \wedge (\forall x.\psi) \models \forall x.(\phi \wedge \psi)$.
3. If ϕ and ψ are first-order formulas and x is a variable, then $\exists x.(\phi \wedge \psi) \models (\exists x.\phi) \wedge (\exists x.\psi)$ and $(\exists x.\phi) \wedge (\exists x.\psi) \models \exists x.(\phi \wedge \psi)$.

It is not encouraged to prepare joint solutions, because we do not support joint exams.